

# **Noise Power Ratio the Analytical Way**

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## **Introduction**

Noise power ratio (NPR) testing is a valuable tool for characterizing return path link performance capability. It provides a quick snapshot of a link's noise and distortion performance with one easy to understand performance curve. Additionally, it is a relatively simple test to perform, particularly with the development of automated NPR test equipment. The development of HFC into a high performance two-way interactive communications medium has meant the need to assure a high quality link. NPR testing is a valuable characterization tool towards meeting this goal.

For the period of time that NPR has been associated with CATV (it has long been associated with other applications, such as satellite communications), the performance has generally been evaluated using manual means. The test is relatively easy, but manual operation can be time consuming. More recently, test equipment has been developed to support NPR testing using an automated measurement run from a PC. This has simplified the test and evaluation process. However, it is always the case from the standpoint of system design to have tools that allow accurate prediction of system performance. With such tools,

it is feasible to explore all of the possibilities in a design via modeling and simulation before committing significant resources to equipment design, integration, and test. Pure computer simulation techniques can be applied, but simulation approaches can have potential for inaccuracy in applications involving nonlinearity, particularly when the nonlinear process cannot itself be simplified with convenient assumptions.

In this paper, we describe an approach that develops a mathematical model that accurately captures the multiple nonlinear impairment sources, providing a complete representation of the composite distortion generation process. Such a model has been used in product development programs to optimize designs, analyze link performance, and predict performance of various HFC architectures and link cascades. Predications are made using a library of equipment models developed consisting of the various link components.

## **NPR Basics**

The idea of NPR can be considered a very straightforward extension of the common two-tone distortion concept. In that case, a device's nonlinear performance is evaluated by driving it

with two CW carriers in its passband, and looking at the device output for new frequency components that did not exist in the input signal. Nonlinear components typically expected are harmonic tones associated with either of the two inputs, as well as mixing products of the two input tones. Observing these undesired components allows determination of the device's intercept points (IP<sub>x</sub>), the fictional point representing the crossing of the distortion input-output curves with the fundamental linear gain curve. In turn, these intercept points are very useful for cascaded system analysis, as the distortion behavior of entire chains can be predicted if the intercept points of the components are known.

The IP<sub>x</sub> concept has generally been associated with narrowband systems. CATV has typically used composite second order (CSO) and composite trip beat (CTB) to characterize nonlinear distortions. These parameters account for the fact that the distortion generation process with a large carrier multiplex creates multiple contributors to second order and third order impairment

Under a reasonable set of assumptions, these composite distortions can be related in a straightforward fashion to the intercept points. For example, in the case of CTB, we can write [9]

$$CTB \text{ (dB)} = 2 [P_c - IP_3] + 10 \text{ Log} (3N^2/8) + 6 \text{ dB}$$

In the above equations, IP<sub>2</sub> and IP<sub>3</sub> represent the second and third order intercept points, respectively. The number of analog (CW) channels is given by N. P<sub>c</sub> is the power per carrier. For this third order case, the equation is

the mid-band CTB, which is the worst case under the assumptions. One such assumption for this to be the case is flat IP<sub>x</sub> performance versus frequency. This is not necessarily a practical assumption for a multi-octave device, and, depending on magnitude, can be important to the accuracy with which performance can be predicted. It is typically handled today with empirical approaches in the forward path. In the reverse path, analysis and measurement as part of the studies undertaken in this paper have shown it to be less significant.

Now, it is often the case that literature or analysis concentrates on third-order distortion, and point out that the third-order intercept point (IP<sub>3</sub>) is the key parameter in evaluating distortion performance. The reason for this bias is that third-order distortion contributions can fall within the signal band even for narrowband systems, the majority of cases for RF systems. By contrast, second order components tend to fall outside of the signal band of interest. However, in HFC return systems, the North American split of 5-40 MHz is three octaves wide. Thus, the second order components can greatly contribute, and this analysis includes those effects.

Extending the concept of two-tone distortion, consider that the actual signal carriage on the return path is generally modulated data-carrying RF signals, applied to the return path at a predetermined power allocation. The reverse path multiplex means that the set of signals applied will each generate distortion components themselves and with one another. Because of the broad passband, the distortion components can fall anywhere in the return band. From

the input signal mix of frequencies, it is predictable where the intermodulation distortion (IMD) components will fall exactly, and how many will land in one frequency location or another using common “beat-mapper” programs.

### **RF and Laser Distortion Contributions**

The ability to estimate the NPR of a system requires the analysis of both its noise and distortion performance. A CATV HFC return link will consist of both RF and optical components. It is convenient to define the optical portion of the link parameters in terms of equivalent RF parameters. This facilitates the mathematical cascading of the link in order to generate the overall intercept points and noise figure, which are then be utilized in the NPR modeling effort. In addition, for optical links, the laser diode clipping characteristics must be known.

The RF link contributes to the cascaded noise figure of the network, as well as to the distortion performance through its 2<sup>nd</sup> and 3<sup>rd</sup> order intercept points (IP2 and IP3). How much of the contribution to the overall link performance is due to the RF section is determined by its relationship to these same parameters in the optical portion of the cascade. In general, the optical link has been the limiting factor in system performance, both in terms of its noise and distortion performance. With improvements in high performance and lower cost optics technology, and the development of more sophisticated RF processing, such as return RF collector architectures and block conversion, the RF section’s performance may take on a larger role in

determining the overall characteristics of a link.

Lasers contribute to NPR limitations in two ways. The first is due to relative intensity noise (RIN), which can be converted into an equivalent noise figure and cascaded with the rest of the system. The second way is through its distortion characteristics, of which there are two main types. One is the familiar RF-like distortion generated due to the non-linearity of its transfer function, which can be characterized by the standard two-tone distortion parameters.

The other key distortion contributor is due to the clipping phenomena that can occur because of the peak-to-average variation of a noise-like return load. This clipping effect can be mathematically evaluated as a non-linear distortion (NLD) parameter [7]. The C/NLD due to clipping usually sets the upper limit of the NPR curve by defining the point at which the carrier to noise plus distortion (C/(N+NLD)) becomes unacceptable, regardless of how good the non-clipping related distortions may be. This curve is also useful, without the RF part of the link, for estimating optimal alignment of optical levels, as can be seen from Figure 1 (note that all figures are located at the end of the text).

### **The Concept of Multitone Distortion**

Estimating the NPR performance of module and cascades using two-tone distortion information requires modification of the resultant intercept points to an approach based on an (ideally) infinite number of infinitesimally spaced tones. Multitone analysis is based on the adjustment of the distortion products as the number of

tones increases, while restricting the total output power level to that of the two-tone level.

In terms of the 3<sup>rd</sup> order products, the resultant distortion level asymptotically approaches a level that is approximately 8 dB higher than that of the two-tone case. This level may be considered the worse case, occurring in the middle of a contiguous band of multitones.

An alternative, but valuable and more computational approach would be to use a beat mapping program in combination with the measured intercept point to calculate the discrete 3<sup>rd</sup> order distortion product levels as they fall over the frequency range of interest. An artificially high number of tones across the band simulates a typical noise density, with the more accurate results occurring with the highest number of tones used. This approach also allows for the estimation of approximate distortion levels over any range of frequencies, instead of just the middle of the signal band, at the expense of computational power. In other words, NPR versus frequency can be analyzed, and this often turns out to be quite useful. For equally spaced tones at equivalent levels, the analysis would show a 3<sup>rd</sup> order product buildup with a shape similar to that of a bell curve centered at the mid-point of the multitone frequencies [8].

For optical systems especially, the 2<sup>nd</sup> order distortion performances may also be a major contributor to the final achievable NPR levels. Using the above logic, it may be deducted that the 2<sup>nd</sup> order level based on an infinite number of tones approaches a point 3 dB higher than the two-tone levels. Care must be

taken, however, in evaluating 2<sup>nd</sup> order distortion for NPR evaluation, due to the frequency dependence of its distortion mapping. The 3 dB point would fall at the location of the maximum 2<sup>nd</sup> order product buildup. Unlike the 3<sup>rd</sup> order situation, this location would depend on the bandwidth, center frequency and spacing of the tones.

Second order distortion patterns consist of two bands of products, each shaped as a triangle with the peaks above and below the frequencies of the original multitones due to the singular addition or subtraction of the tones [8]. The maximum number of products corresponding to the 3 dB point mentioned earlier occurs at the peak of the lower triangle, whose frequency is a function of the spacing of the multitones. This peak occurs at DC. It is necessary to identify the frequency of interest in the triangle to find the actual level of the 2<sup>nd</sup> order distortion contribution. For a 5-40 MHz return path bandwidth, the mid-band multitone 2<sup>nd</sup> order distortion level is approximately the level of the initial two-tone distortions and is a function of both the lower *and* upper product triangles. Thus, for analysis that targets midband 5-40 MHz performance, it is sufficient to use the two-tone distortion level for this portion of the NPR contribution.

### **NPR Mathematical Modeling**

The procedure for generating NPR estimates involves evaluating the noise and distortion performance of the system. The NPR curve itself is a measure of carrier-to-noise-plus-nonlinear distortion ratio,  $C/(N+NLD)$ , where NLD in this cases considers both that due to the optics and the

intermodulation distortion (IMD) of the RF nonlinearity. For NPR, the desired signal, C, is represented by a noise source.

The overall C/N must be found to characterize the link noise performance. Noise parameters may be specified or measured on the individual components of a link or on an entire link. Components of the link may have a guaranteed noise performance. However, depending upon the component, the noise parameter used to characterize the device may vary. The optical/RF transducers are the obvious examples of such components. But an equivalent NF can always be generated, and can be determined directly from the circuit design if the design is known. Following this conversion to NF, cascaded NF analysis common to system analysis can be used. The relationship between C/N of a component or link and its equivalent NF is simply

$$C/N(\text{dB}) = P_s(\text{dBW}) - \{[10\text{Log}(k \cdot 290 \cdot \text{BW})] + \text{NF}(\text{dB}) + \text{Gain}(\text{dB})\}, \quad (1)$$

for signal power,  $P_s$ , Boltzmann constant  $k$ , and system bandwidth,  $\text{BW}$ .

For the distortion characterization required to develop NPR, the single second order (SSO) intermodulation distortion (IMD2) is found in order to calculate C/IMD2. Single third order (STO) distortion is also found, to calculate C/IMD3. A key step is the adjustment of SSO and STO values to account for noise signals versus tone analysis by extrapolating two-tone theory to noise-like signals using multitone theory previously mentioned [5]. The resultant C/IMD2 and C/IMD3

are added power-wise to get a composite C/IMD for the RF distortions.

It has been found through modeling and verification that RF distortion products up to third order only is necessary to develop accurate NPR models. As with the noise case, distortion can be measured or specified at the component or link level to use in predicting NPR performance. Cascade analysis applies when information is known at the component level, using well-known cascaded intercept point relationships. Individual intercept points can be found from the following well known two-tone distortion relationships:

$$\text{SSO} = \text{IP2} - P_{\text{tone}} \quad (2)$$

$$\text{STO} = 2 [\text{IP3} - P_{\text{tone}}], \quad (3)$$

for per-tone power  $P_{\text{tone}}$ . Note that all intercept point references and analysis in this paper refer to the output intercept point of the component or the cascade.

For adjustment between tone distortion and NPR, modeled via multitone analysis, the midband C/IMD3 becomes about 8 dB worse as previously described (actually 7.7 dB). By contrast, for this case, the second order midband response does not change. However, this is not the case across the entire, multi-octave return. This can be seen in modeling and measuring NPR versus frequency.

When there is an optical link, as in the HFC case, the clipping C/NLD and  $C/N_{\text{in}}$  (Carrier-to-noise due to interferometric intensity noise), if necessary, are included. The C/NLD contribution can be calculated [2][6][7]. The NLD due to clipping does account

for non-negligible higher order distortion products that this mechanism can generate. The total NLD due to clipping is a function of the rms optical modulation index (OMI) used to drive the laser. C/NLD can be approximated as

$$C/NLD = 10\text{Log}\{[(.4U^3)/(1+6U^2)]\exp(-1/2U^2)\}, \quad (4)$$

where

$$U = (\text{OMI per ch}) \cdot \sqrt{(N/2)}.$$

The OMI is, of course, tied to the RF drive level at the point in the cascade where the link converts the RF to linear optics. Thus, as with typical distortion contributors, C/NLD varies as a function of RF drive level just as C/IMD2 and C/IMD3 do.

For optical networks that use DFB lasers and go through appreciable link lengths (>10 dB loss, for example), it may be necessary to include the interferometric intensity noise generated by the laser/fiber interface. The procedures and equations necessary to calculate this effect, and the theory behind them, is also available [1][4].

The RF and optical distortions are combined to generate the total distortion power. The noise and distortion contributions create a composite C/(N+NLD). This process is repeated over a range of RF levels at the front of the cascade to sweep the complete NPR curve.

A library of component models and their various noise and distortion parameters allows prediction of NPR performance across a variety of architectures.

## **Bandwidth and Frequency Dependence**

This paper concerns itself primarily with the estimation of NPR at the mid-band of a 5 to 40 MHz return band. However, NPR modeling estimations are in fact frequency dependent. This is largely due to the product mapping discussed previously of the 2<sup>nd</sup> and 3<sup>rd</sup> order distortions. The frequency dependence is secondarily due to the possible frequency variations associated with the numerous noise contributors.

The 3<sup>rd</sup> order product mapping results in a bell-shaped response with the peak occurring at mid-band of the tone generating frequencies. The distortion level roll off at the edges of the generating band has been investigated previously [8][9] and has been shown to be approximately 1.76 dB lower than that of the mid-band response.

As shown in [8], the 2<sup>nd</sup> order products resemble two triangle patterns that may or may not fall partially in the band of interest. The location of the additive products for the NPR models will begin at twice the starting frequency and end at twice the stop frequency with the maximum density occurring at the start plus stop frequencies. The subtractive products are found from DC (the triangle is completed in the negative frequency realm) to the stop frequency minus the start frequency. As can be seen for the 5-40 MHz return, because of the multi-octave nature of the passband, some resultant 2<sup>nd</sup> order products do fall within the generation band and should be included in any estimation of NPR.

There may be instances when the products fall outside the band of interest, such as a forward path with a link dedicated to carrying digitally modulated RF only between 550-870 MHz. Under such circumstances the NPR may be estimated without the contributions due to 2<sup>nd</sup> orders. It must be kept in mind that, while this may indeed be true in evaluating the particular digital channels, the out-of-band 2<sup>nd</sup> order products can cause interference problems with standard forward analog channels. Unless filtering is applied, this may be the case when the analog channels are located below the digital at a point where the transport link becomes a composite signal load of analog and digital.

For optical networks there would also be secondary frequency dependent effects to NPR that are due to the laser and fiber phenomena themselves as well as the interactions between the two. Both interferometric intensity noise (iin) and phase to intensity noise are frequency dependent and would have to be evaluated as such in any widely swept frequency analysis of NPR. Typically, phase to intensity noise is a negligible effect and can be ignored for modeling purposes. Finally, practical cascaded systems can also incur a thermal noise floor variation over frequency.

### **Analysis vs. Measurement**

Using the modeling approach discussed, we can compare estimated NPR performance of various return path systems with measured results.

The first system consists of an unisolated Fabry-Perot (FP) 0.4 mW laser

transmitter operating through 25 km of fiber into an optical receiver with a 9.7 pA/√Hz noise current. Optical loss associated with the 25 km of fiber at 1310 nm was measured at 9 dB, close to the expected .35 dB/km. The transmitter is a Motorola model SG1-FPT, one of a family of node transmitters available for return path applications. The link terminates into a Motorola return path optical receiver, model AM-OMNI-RPR/2C. The signal loading for the transmitter consists of 35 MHz of noise (5-40 MHz) at total power levels from 7 dBmV to +30 dBmV. The recommended operating total signal level for the transmitter is +20 dBmV.

Measurements of the FP diodes used in the transmitter show a typical relative intensity noise (RIN) of -132 dB/Hz. This is an equivalent RF NF of 42.8 dB for an efficiency of 0.1 W/A. When modeled with the RF portion of the transmitter, this resulted in an overall noise figure of 26.6 dB. Two tone distortion data for the diodes showed average 2<sup>nd</sup> order levels of -53.4 dBc and 3<sup>rd</sup> order levels of -64.6 dBc. After cascading this with the RF portion of the transmitter, the resultant overall levels dropped to -51.7 dBc and -64.5 dBc respectively. Based on these numbers, the calculated intercept points were +65.1 dBmV for 2<sup>nd</sup> order (IP2) and +45.7 dBmV for 3<sup>rd</sup> order (IP3).

The optical receiver RPR/2C uses a photodiode with a typical responsivity of 0.85 A/W. The internal gain control was set for the mid-gain of its 20 dB range, resulting in the measured EINC of 9.7 pA/√Hz. This corresponds to an equivalent noise figure of 4.5 dB at an optical input of -13.0 dBm. Typical measured 2<sup>nd</sup> and 3<sup>rd</sup> order levels of the

receiver at 0 dBm optical input and 20% OMI per tone at the transmitter are -65 dBc and -75 dBc respectively. This results in an IP2 of 117.0 dBmV and an IP3 of 89.5 dBmV.

The transmitter and receiver were cascaded through the 25 km fiber (RF loss of 18 dB) and the overall electrical parameters for the chain were generated for use in the NPR estimations. The link is shown in Figure 2.

Choosing a single point of operation, for an input to the transmitter of 20 dBmV, the resultant C/N, distortions, and final NPR were found as follows:

$C/N = 40.5 \text{ dB}$   
 $IP3 = 64.3 \text{ dBmV}; C/IMD3 = 56.8 \text{ dB}$   
 $IP2 = 83.5 \text{ dBmV}; C/IMD2 = 51.5 \text{ dB}$   
 $C/NLD = 70.4 \text{ dB}$   
 $Total \text{ NPR @ } 20 \text{ dBmV} = 40.1 \text{ dB}.$

Now, this being the recommended operating point, it is expected that linear operation would be the situation here, as can be seen by the dominance of C/N to the resultant NPR. Of course, for a complete NPR curve, the above calculation is performed at every drive level. The modeled mid-band NPR curve based on the cascaded NF and intercept points added to a carrier-to-nonlinear-distortion C/NLD curve of the clipping generates the overall modeled NPR curve shown in Figure 3. The example link was then measured for NPR using the Noise-Com automated NPR Test Station. These results are also shown in Figure 3 for comparison. The modeled versus measured data show good correlation.

A second return system utilizing a 1 mW distributed feedback (DFB) laser diode

was also evaluated. The transmitter used was a 1 mW DFB, Motorola model SG2-DFBT. The RPR/2C was again used as the receiver. By using the same procedures as outlined for the SG1-FPT, the overall cascaded electrical results for this system are shown in Figure 4. The resultant model and measured NPR curves are shown in Figure 5. Again, the plots show good correlation. In addition to the curves generated by the equivalent RF electrical parameters and the diode clipping, the overall model for the DFB link also includes the interferometric intensity noise (iin) contribution due to the scattering effects of the laser/fiber interface.

As the return path is slowly transitioned from analog to digital, the ability to estimate the NPR performances of systems utilizing analog-to-digital converters (A/D's) becomes important. This technique can also be used to incorporate the A/D's quantization and saturation noise performance into an NPR response for a complete digital link. Figure 6 shows the modeled and measured performances of a 10-bit A/D digital return system. A 10-bit system has roughly the same performance as linear optics with DFB lasers. A full discussion of the theory behind evaluating digital links and their analogy to analog transmission types may be found in [3].

## Conclusion

NPR has become a valuable tool for characterizing return path performance capability. It provides, in one easy snapshot, both noise and distortion characteristics of a complete link. In addition, it is a simple test to setup and make measurements on. The drawback

with NPR since it has come into the realm of CATV has been that system designers have been constrained to some extent by having to rely on measurements. Measurement requires time and resources each time a system design verification is desired. Simulations put the designer at the mercy of the programming code and the potential for inaccuracy in dealing with nonlinearity. In this paper, we have presented a mathematical approach extrapolated from the well-understood distortion analysis of both RF and optical systems. In this approach, the two subsystems can be integrated in a single spreadsheet analysis that can accurately predict NPR. This provides system designers the tool they need to characterize new designs and new architectures, develop performance specifications to the component level, and yield deeper insight into the mechanism of noise power distortion effects.

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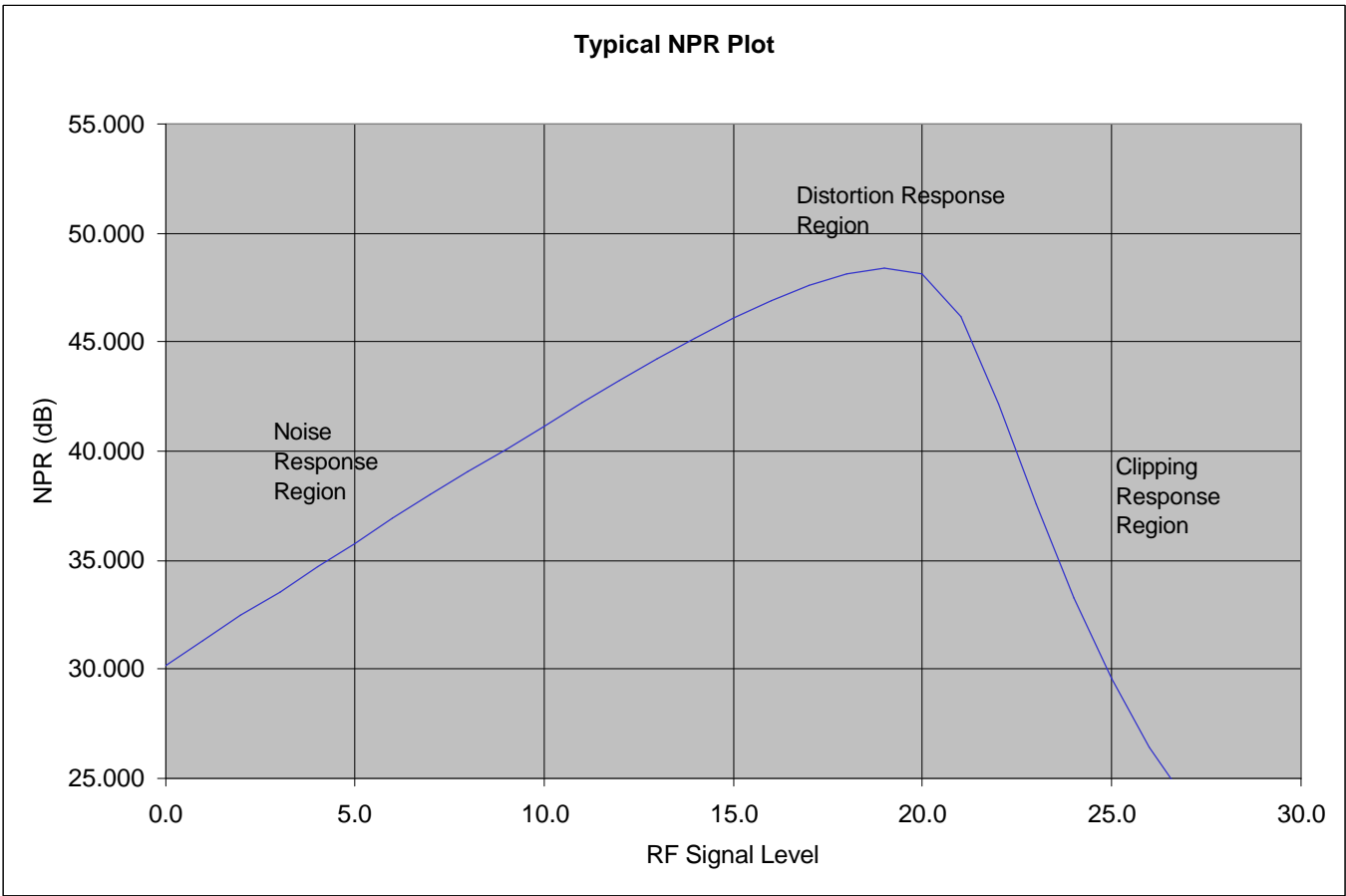
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### **Acknowledgements**

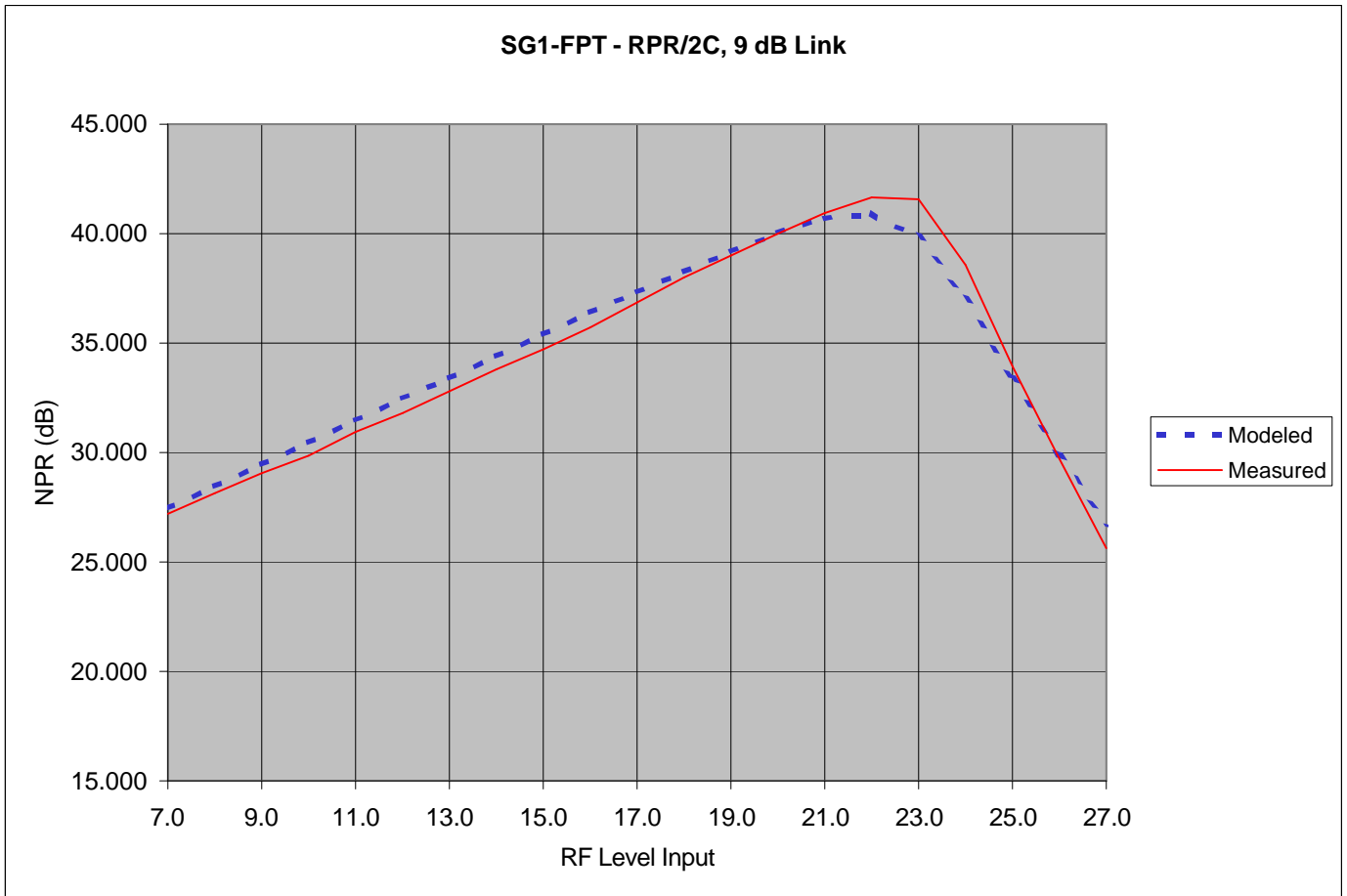
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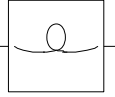

**Figure 1 – Typical NPR Plot**

	SG1-FPT (FO TX)	Fiber Loss (RF)	RPR/2C (FO RX)	Cascade Total
Gain (dB)	-3.59	-18	36.59	15.00
NF (dB)	26.55	18	4.47	29.31
Output 1 dB Comp dBmV	300	300	300	299.94
Output IP3 (dBmV)	45.68	300	89.5	64.26
Output IP2 (dBmV)*	65.06	300	117	83.47

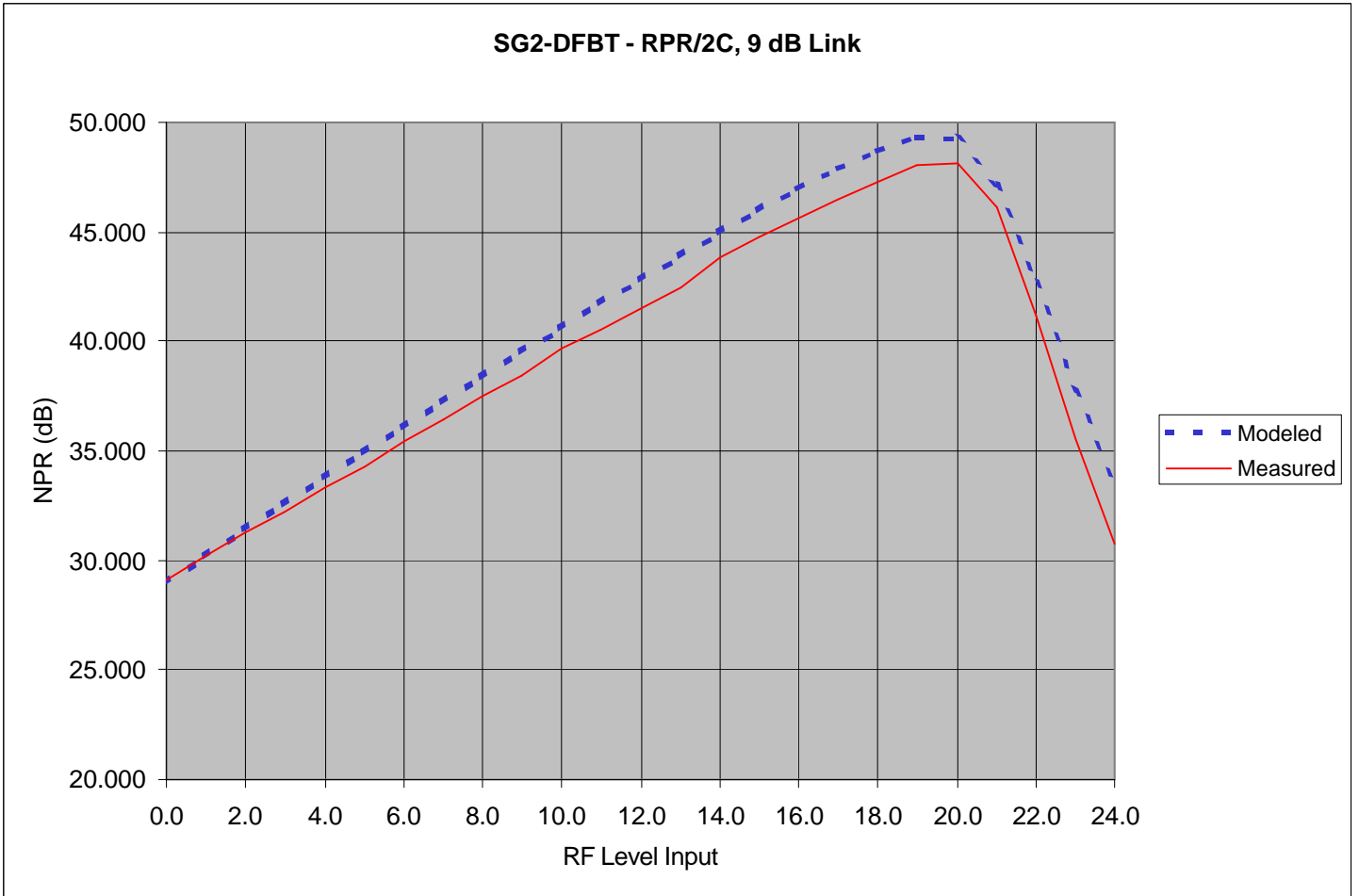
**Figure 2 – Cascade Analysis for a 9 dB Fabry-Perot (FP) Return Link**



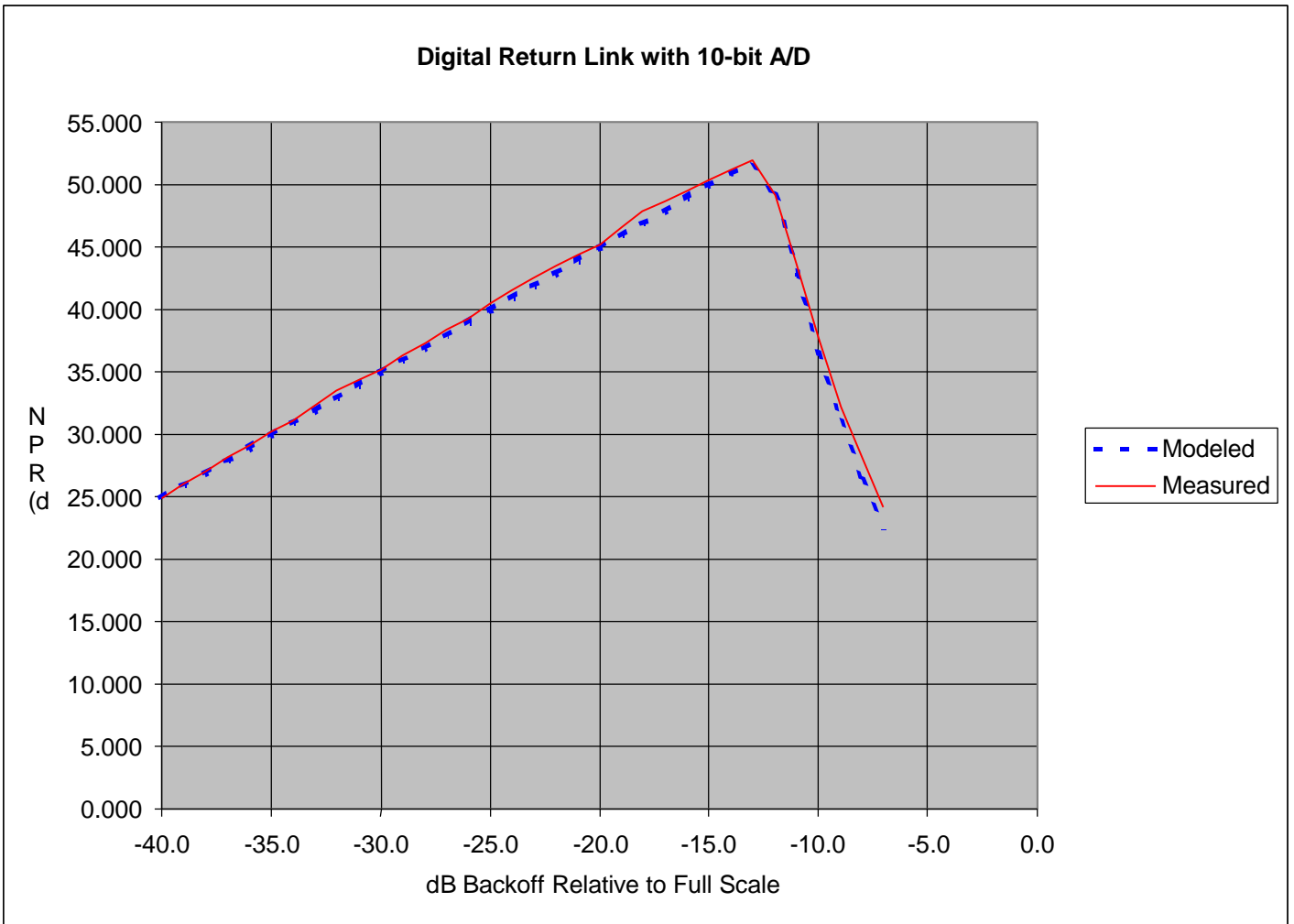
**Figure 3 – NPR Plot for a 9 dB Fabry-Perot (FP) Return Link**

	SG2-DFBT	Fiber Loss (RF)	AM-OMNI-RPR/2C	Cascade Total
				
<b>Gain (dB)</b>	5.51	-18	36.59	<b>24.10</b>
<b>NF (dB)</b>	6.26	18	5.23	<b>18.00</b>
<b>Output 1 dB Comp dBmV</b>	300	300	300	<b>299.94</b>
<b>Output IP3 (dBmV)</b>	55.51	300	89.5	<b>73.98</b>
<b>Output IP2 (dBmV)*</b>	81.16	300	117	<b>98.63</b>

**Figure 4 – Cascade Analysis for a 9 dB Distributed Feedback (DFB) Return Link**



**Figure 5 – NPR Plot for a 9 dB Distributed Feedback (DFB) Return Link**



**Figure 6 – NPR Plot for a 10-bit Digital Return Link**